Solución al problema de "Resuelve esta otra integral definida"

Enunciado:

$$\int_9^2 (2f(x)+3)\,dx = -17 \,, \int_2^5 \frac{f(x)}{3}\,dx = -2 \quad , \text{¿} \int_5^9 (4f(x)-1)\,dx \text{?}$$

Solución:

$$\int_5^9 (4f(x)-1)\,dx = 4\int_5^9 f(x)\,dx - \int_5^9 1\,dx = 4\cdot\left(\int_5^9 f(x)\,dx - 1\right)$$

$$-2 = \int_2^5 \frac{f(x)}{3}\,dx = \frac{1}{3}\int_2^5 f(x)\,dx \Rightarrow \int_2^5 f(x)\,dx = -6$$

$$-17 = \int_9^2 (2f(x)+3)\,dx = -\int_2^9 (2f(x)+3)\,dx \Rightarrow \int_2^9 (2f(x)+3)\,dx = 17$$

Pero $17 = \int_2^9 (2f(x)+3)\,dx = 2\int_2^9 f(x)\,dx + \int_2^9 3\,dx = 2\int_2^9 f(x)\,dx + 21 \Rightarrow \int_2^9 f(x)\,dx = -2$

Ahora bien:

$$\int_2^9 f(x)\,dx = \int_2^5 f(x)\,dx + \int_5^9 f(x)\,dx \Rightarrow \int_5^9 f(x)\,dx = \int_2^9 f(x)\,dx - \int_2^5 f(x)\,dx =$$

$$= \int_5^9 f(x)\,dx = -2-(-6) = 4$$

Finalmente: $\int_5^9 (4f(x)-1)\,dx = 4\cdot\left(\int_5^9 f(x)\,dx - 1\right) = 4\cdot(4-1) = \mathbf{12}$

Solución: *12*