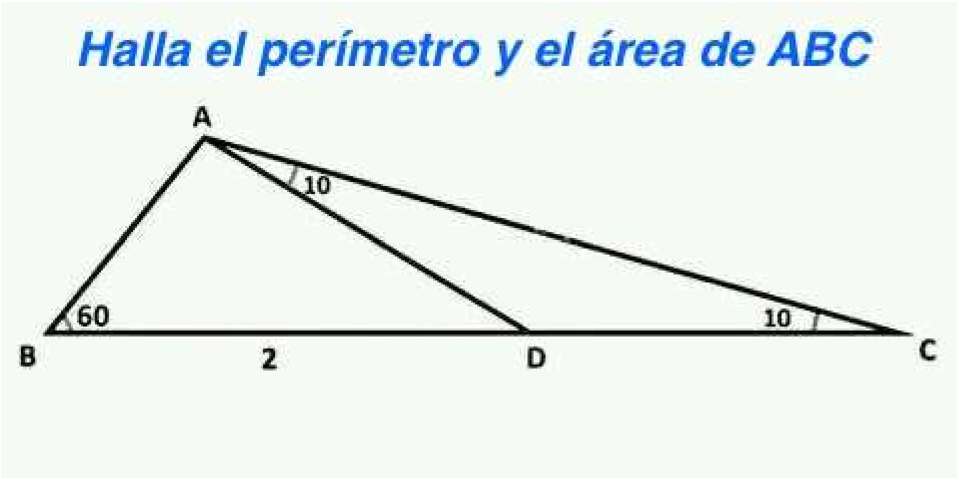
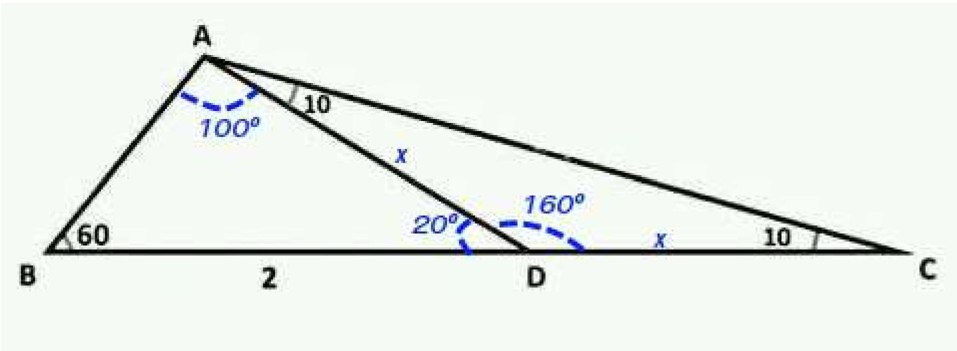
Solución a "Halla el perímetro y el área de ABC"

Enunciado:



Solución:

Consideremos la figura con estos datos (obvios):



En el triángulo *ABD* : $\frac{x}{sen\,60^o}=\frac{2}{sen\,100^o}\Rightarrow x=\frac{2\cdot sen\,60^o}{sen\,100^o}=\frac{\sqrt{3}}{sen\,100^o}$ y

$\frac{AB}{sen\,20^o}=\frac{2}{sen\,100^o}\Rightarrow AB=\frac{2\cdot sen\,20^o}{sen\,100^o}$

En el triángulo *ACD* : $\frac{AC}{sen\,160^o}=\frac{x}{sen\,10^o}\Rightarrow AC=\frac{sen\,160^o}{sen\,10^o}\cdot\frac{\sqrt{3}}{sen\,100^o}=\frac{\sqrt{3}\cdot sen\,160^o}{sen\,10^o\cdot sen\,100^o}$

Perímetro de ***ABC***:

$$P_{ABC}=AB+BC+CA=\frac{2\cdot sen\,20^o}{sen\,100^o}+2+\frac{\sqrt{3}}{sen\,100^o}+\frac{\sqrt{3}\cdot sen\,160^o}{sen\,10^o\cdot sen\,100^o}$$

$$P_{ABC}=\frac{2\cdot sen\,10^o\cdot sen\,20^o+2\cdot sen\,10^o\cdot sen\,100^o+\sqrt{3}\cdot sen\,10^o+\sqrt{3}\cdot sen\,160^o}{sen\,10^o\cdot sen\,100^o}$$

$$P_{ABC}=\frac{2\cdot sen\,10^o\cdot(sen\,20^o+sen\,100^o)+\sqrt{3}\cdot(sen\,10^o+sen\,160^o)}{sen\,10^o\cdot sen\,100^o}$$

$$P_{ABC}\approx 7'9175$$

El área sería:

$$A_{ABC}=\frac{1}{2}\cdot AB\cdot BC\cdot sen\,60^o=\frac{1}{2}\cdot\frac{2\cdot sen\,20^o}{sen\,100^o}\cdot\left(2+\frac{\sqrt{3}}{sen\,100^o}\right)\cdot\frac{\sqrt{3}}{2}$$

$$A_{ABC}=\frac{\sqrt{3}\cdot sen\,20^o\cdot(2\cdot sen\,100^o+\sqrt{3})}{2\cdot(sen\,100^o)^2}$$

$$A_{ABC}\approx 1'1305$$

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