Solución a "Find the requested sum"

Enunciado:

$$if\ \ 3^x+3^y=10\ \ and\ \ 3^{x+y}=5$$

$$find: 3^{x-y}+\frac{1}{3^{x-y}}$$

Solución:

$$3^{x-y}+\frac{1}{3^{x-y}}=\frac{3^x}{3^y}+\frac{1}{\frac{3^x}{3^y}}=\frac{3^x}{3^y}+\frac{3^y}{3^x}=\frac{3^{2x}+3^{2y}}{3^x\cdot 3^y}=\frac{\left(3^x\right)^2+\left(3^y\right)^2}{3^{x+y}}=\frac{\left(3^x\right)^2+\left(3^y\right)^2}{5}$$

Por otro lado:

$$3^x+3^y=10\Rightarrow 100=\left(3^x+3^y\right)^2=\left(3^x\right)^2+\left(3^y\right)^2+2\cdot 3^x\cdot 3^y=\left(3^x\right)^2+\left(3^y\right)^2+2\cdot 3^{x+y}=\left(3^x\right)^2+\left(3^y\right)^2+10$$

Por lo que: $\left(3^x\right)^2+\left(3^y\right)^2=90$

Sustituyendo obtenemos: $3^{x-y}+\frac{1}{3^{x-y}}=\frac{90}{5}=18$

Solución: *18*

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